

 $\begin{array}{l} \mbox{Thm: Sum Theorem for Dimension n:} \\ \mbox{If } X = \cup_{i=1}^{\infty} F_i \mbox{ where the } F_i \mbox{ are closed in } X \\ \mbox{and } ind(F_i) \leq n, \mbox{ then } ind(X) \leq n. \end{array}$

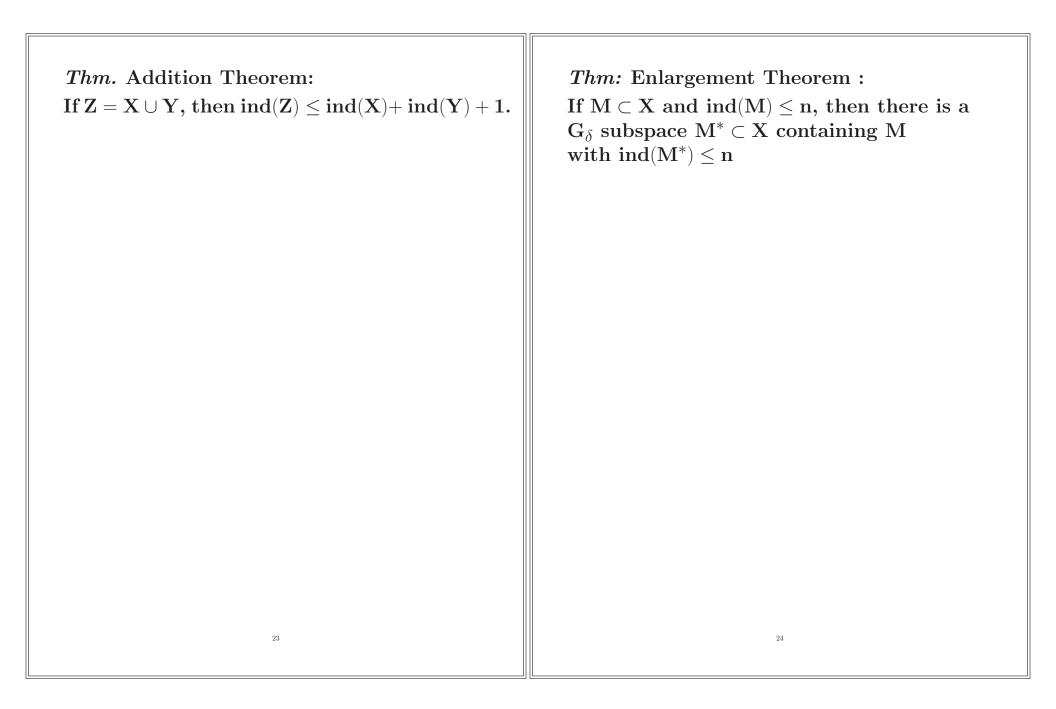
Cor: If $X = \bigcup_{i=1}^{\infty} F_i$ where the F_i are F_{σ} sets in X and $ind(F_i) \leq n$, then $ind(X) \leq n$.

Cor: If X is the union of $\leq n$ dimensional subspaces A and B where A is closed in X, then $ind(X) \leq n$.

Cor: If X is the union of a $\leq n$ dimensional subspace A and a finite set B, then $ind(X) \leq n$.

 $\begin{array}{l} \textit{Thm:} \ First \ Decomposition \ Theorem: \\ X \ satisfies \ ind(X) \leq n \ if \ and \ only \ if \ X \ can \ be \\ represented \ as \ a \ union \ of \ subspaces \ Y \ and \\ Z \ where \ ind(Y) \leq n-1 \ and \ ind(Z) \leq 0. \end{array}$

 $\begin{array}{l} \mbox{Thm: Second Decomposition Theorem :} \\ X \ satisfies \ ind(X) \leq n \ if \ and \ only \ if \ X \ can \\ be \ represented \ as \ a \ union \ of \ n+1 \ subspaces \\ Z_1, \ldots Z_{n+1} \ such \ that \ for \ each \ i, \ ind(Z_i) \leq 0. \end{array}$



Thm: First Separation Theorem :

If $ind(X) \leq n,$ then for every pair A,B of closed disjoint subspaces of X, there is a partition L between A and B such that $ind(L) \leq n-1$

Thm: Second Separation Theorem :

If $ind(M) \leq n$, and $M \subset X$, then for every pair A, B of closed disjoint subspaces of X, there is a partition L between A and B such that $ind(L \cap M) \leq n - 1$

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Subspaces

 $\begin{array}{l} \textit{Thm:} A \ subspace \ M \subset X \ satisfies \ ind(M) \leq n \\ \textit{if and only if} \\ \forall x \in X \ and \ \forall \ open \ set \ V \subset X \ with \ x \in V, \\ \exists \ an \ open \ set \ U \subset X \ with \ x \in U \subset V \ and \\ with \ ind(M \cap Bd(U)) \leq n-1 \\ \textit{if and only if} \end{array}$

 ${\bf X}$ has a countable basis ${\bf B}$ such that $ind({\bf M}\cap Bd({\bf U}))\leq n-1$ for each ${\bf U}\in {\bf B}$.

 $\begin{array}{l} \mbox{Thm: Cartesian Product Theorem:} \\ \mbox{If X and Y are nonempty,} \\ \mbox{ind}(\mathbf{X}\times\mathbf{Y}) \leq \mbox{ind}(\mathbf{X}) {+} \mbox{ind}(\mathbf{Y}) \end{array}$

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