Dimension n Results:

Theorem: **If ind(X)= $n \geq 1$, then for each $k < n$, X contains a closed subspace M_k with $\text{ind}(M_k) = k$**

Lemma: **If X can be represented as the union of subspaces Y and Z, with $\text{ind}(Y) \leq n - 1$ and $\text{ind}(Z) \leq 0$, then $\text{ind}(X) \leq n$.**

Thm: **Sum Theorem for Dimension n:**
If $X = \cup_{i=1}^{\infty} F_i$ where the F_i are closed in X and ind(F_i)$\leq$ n, then ind(X)$\leq$ n.

Cor: **If $X = \cup_{i=1}^{\infty} F_i$ where the F_i are F_σ sets in X and ind(F_i) $\leq$ n, then ind(X)$\leq$ n.**

Cor: **If X is the union of $\leq n$dimensional subspaces A and B where A is closed in X, then ind(X)$\leq$ n.**

Cor: **If X is the union of a $\leq n$ dimensional subspace A and a finite set B, then ind(X)$\leq$ n.**

Thm: **First Decomposition Theorem :**
X satisfies ind(X) $\leq$ n if and only if X can be represented as a union of subspaces Y and Z where ind(Y) $\leq$ n $-$ 1 and ind(Z) $\leq$ 0.

Thm: **Second Decomposition Theorem :**
X satisfies ind(X) $\leq$ n if and only if X can be represented as a union of n + 1 subspaces $Z_1, \ldots Z_{n+1}$ such that for each i, ind(Z_i) $\leq$ 0.

Thm. **Addition Theorem:**

If $\mathbf{Z} = \mathbf{X} \cup \mathbf{Y}$, then $\mathbf{ind}(\mathbf{Z}) \leq \mathbf{ind}(\mathbf{X}) + \mathbf{ind}(\mathbf{Y}) + \mathbf{1}$.

Thm: **Enlargement Theorem :**

If $\mathbf{M} \subset \mathbf{X}$ and $\mathbf{ind}(\mathbf{M}) \leq \mathbf{n}$, then there is a $\mathbf{G}_\delta$ subspace $\mathbf{M}^* \subset \mathbf{X}$ containing $\mathbf{M}$ with $\mathbf{ind}(\mathbf{M}^*) \leq \mathbf{n}$

Thm: **First Separation Theorem :**

If $\mathbf{ind(X) \leq n}$, then for every pair $\mathbf{A, B}$ of closed disjoint subspaces of X, there is a partition L between A and B such that $\mathbf{ind(L) \leq n - 1}$

Thm: **Second Separation Theorem :**

If $\mathbf{ind(M) \leq n}$, and $\mathbf{M \subset X}$, then for every pair $\mathbf{A, B}$ of closed disjoint subspaces of X, there is a partition L between A and B such that $\mathbf{ind(L \cap M) \leq n - 1}$

Subspaces

Thm: **A subspace $M \subset X$ satisfies $\text{ind}(M) \leq n$**

if and only if

$\forall x \in X$ and $\forall$ open set $V \subset X$ with $x \in V$, $\exists$ an open set $U \subset X$ with $x \in U \subset V$ and with $\text{ind}(M \cap \text{Bd}(U)) \leq n - 1$

if and only if

X has a countable basis B such that $\text{ind}(M \cap \text{Bd}(U)) \leq n - 1$ for each $U \in B$.

Thm: **Cartesian Product Theorem:**

If X and Y are nonempty, $\text{ind}(X \times Y) \leq \text{ind}(X) + \text{ind}(Y)$