Def. **The large inductive dimension of a separable metric space X, Ind(X), is defined as follows:**

- **Ind(X) $= -1$ if and only if X $= \emptyset$**
- **Ind(X) $\leq$ n, for n ≥ 0, if for every closed set A $\subset$ X and for each open set V $\subset$ X containing A, there exists an open set U $\subset$ X such that A $\subset$ U $\subset$ V with Ind(Fr(U)) $\leq$ n $-$ 1**
- **Ind(X) $=$ n if Ind(X) $\leq$ n and Ind(X) $>$ n $-$ 1**
- **Ind(X) $= \infty$ if Ind(X) $>$ n for all n.**

Note: **Equivalently, Ind(X) $\leq$ n if and only if for each pair of closed disjoint sets A and B in X, there is a partition L between A and B with Ind(L) $\leq$ n $-$ 1**

Theorem: **For each X, ind(X) $\leq$IndX.**

Theorem: **For each X, ind(X) $=$Ind(X).**

Definition: **The *order* of a collection A of subsets of X is the largest integer n such that the family contains $n+1$ subsets with nonempty intersection. The order is ∞ if no such integer exists.**

Definition: **The covering dimension of X, $\dim(X)$, is defined as follows:**

- **$\dim(X) \leq n$, for $n \geq -1$, if every finite open cover of the space X has a finite open refinement of order $\leq n$**
- **$\dim(X) = n$ if $\dim(X) \leq n$ and $\dim(X) > n-1$**
- **$\dim(X) = \infty$ if $\dim(X) > n$ for all n.**

Note: **Recall definitions of refinement and shrinking of covers.**

Theorem: **The following are equivalent:**

- **$\dim(X) \leq n$,**
- **Every finite open cover of X has an open refinement of order $\leq n$, and**
- **Every finite open cover of X has an open shrinking of order $\leq n$**

Theorem: **$\dim(X) \le n$ if and only if every $n+2$ element open cover has an open shrinking of order $\le n$**

Theorem: **$\dim(X) \le n$ if and only if every $n+2$ element open cover has an open shrinking of order $\le n$**

Corollary: **$\dim(X) = 0$ if and only if $\mathrm{Ind}(X) = 0$**

Theorem: **For a compact metric $(\mathbf{X}, \mathbf{d})$, the following are equivalent:**

- **$\dim(\mathbf{X}) \leq \mathbf{n}$**
- **For every metric ρ on X and $\epsilon > 0$ there is a finite open cover of X of order $\leq \mathbf{n}$ and of mesh $< \epsilon$**
- **There is a metric ρ on X such that for all $\epsilon > 0$ there is a finite open cover of X of order $\leq \mathbf{n}$ and of mesh $< \epsilon$**