Next, we will show $\dim(X) \leq \mathrm{ind}(X)$

Lemma: **For every family $\{F_i | 1 \leq i \leq k\}$ of pairwise disjoint closed subsets of a subspace M of X, there is a family $\{W_i | 1 \leq i \leq k\}$ of open subsets of X with $F_i \subset W_i$.**

Theorem: **$\dim(X) \leq \mathrm{ind}(X)$.**

For $\mathrm{ind}(X) \leq \dim(X)$

- One approach is to prove this is true for compact X and then prove a compactification theorem for dim using completion of a certain metric.

- A second approach is to embed X in $M_n^{2n+1} \cap I^{2n+1} = \left(\bigcup_{i=0}^{n} Q_i^{2n+1}\right) \cap I^{2n+1}$ in such a way that $\overline{X} \subset M_n^{2n+1}$

Recall, For Y compact, Y^X with the sup metric is a complete metric space, and thus has the Baire property. That is, a countable intersection of dense open sets is dense.

Def. Let X be compact and $\epsilon > 0$. A map $g : X \to Y$ is an ϵ mapping if the inverse image and any point in Y had diameter $< \epsilon$.

Lemma: Let X be compact. A surjective map $g : X \to Y$ is a homeomorphism if and only if it is a $1/i$ map for each i.

Lemma: Let X be compact. Then for each $\epsilon > 0$, the set G_ϵ of ϵ mappings is open in Y^X

Lemma: Let X be compact, with $\dim(X) \leq n$. Let L be fixed n - dimensional hyperplane in R^{2n+1}. For $\epsilon > 0$, let G_ϵ^L be the set of ϵ mappings in $(I^{2n+1})^X$ with image missing L. Then G_ϵ^L is dense and open open in $(I^{2n+1})^X$.

Theorem: Let X be compact, with $\dim(X) \leq n$ Then the set of mappings in $(I^{2n+1})^X$ with image in M_n^{2n+1} is dense in $(I^{2n+1})^X$.

Corollary: Let X be compact, with $\dim(X) \leq n$. Then $\text{ind}(X) \leq n$.