Name: ______________________________

1. A population of fish being harvested is modeled by $dp/dt = 2p - 100$, where $p > 0$ is the population. Suppose that initially the population $p_o \neq 50$. As time increases, the population

   A. converges to $p = 50$

   B. diverges from $p = 50$

   C. goes to zero

   D. diverges from $p = 50$ if $p_0 < 50$, otherwise converges

   E. diverges from $p = 50$ if $p_0 > 50$, otherwise converges

2. The first order linear equation $y' - \sin(t)\, y = 4 - t$ admits the integrating factor

   A. $\mu(t) = \mathrm{e}^{-\sin(t)}$

   B. $\mu(t) = \mathrm{e}^{\sin(t)}$

   C. $\mu(t) = \mathrm{e}^{-\cos(t)}$

   D. $\mu(t) = \mathrm{e}^{\cos(t)}$

   E. $\mu(t) = \mathrm{e}^{-t\sin(t)}$

3. The equation

$$\frac{d^2y}{dx^2} = y^3$$

   can best be described as a

   A. second order, nonlinear ODE

   B. third order, linear ODE

   C. first order, linear ODE

   D. third order, nonlinear PDE

   E. second order, linear PDE

4. Given the differential equation

$$ty' - y = t^2 \mathrm{e}^{-t}, \quad t > 0$$

the general solution is

A. $y = -\left(2 + 2\,t + t^2\right)\mathrm{e}^{-t} + ct$

B. $y = -t\mathrm{e}^{-t} + ct$

C. $y = -ct\mathrm{e}^{-t}$

D. $y = -\mathrm{e}^{-t} + c$

E. $y = -c\mathrm{e}^{-t}$

5. Given the initial value problem

$$y' = \frac{1}{2}y^2 - xy^2, \quad y(0) = 2$$

the solution is

A. $y = \dfrac{1}{\frac{1}{2}x^2 - x + \frac{1}{2}}$

B. $y = \dfrac{2}{x^2 + \frac{1}{2}x + 1}$

C. $y = \dfrac{2}{x^2 - x + 1}$

D. $y = \dfrac{1}{x^2 - x + \frac{1}{2}}$

E. $y = \dfrac{1}{\frac{1}{2} + x - x^2}$

6. The initial value problem $y' + t^3 y = \tan^2 t$, $y(0) = 2$

   A. has a solution which is defined everywhere, but the solution is not unique

   B. has a unique solution which is defined everywhere

   C. has a solution on the interval $-\pi/2 < t < \pi/2$, but the solution is not unique

   D. does not have a solution

   E. has a unique solution, but this solution only exists on the interval $-\pi/2 < t < \pi/2$

7. A toxic chemical has been spilled into a lake with volume V gallons. Fresh water flows into the lake at rate r gallons/day, the flow out of the lake is at the same rate. Assuming the chemical is evenly distributed throughout the lake, a model for the amount of chemical Q in the lake is given by

   A. $\dfrac{dQ}{dt} = -\dfrac{rQ}{V}$

   B. $\dfrac{dQ}{dt} = \dfrac{r}{V} - \dfrac{rQ}{V}$

   C. $\dfrac{dQ}{dt} = -rQ$

   D. $\dfrac{dQ}{dt} = -rQV$

   E. $\dfrac{dQ}{dt} = rV - rQ$

8. The differential equation $(3xy + y^2) + (x^2 + xy)y' = 0$, which is not exact, admits the integrating factor

   A. $\mu(y) = y$

   B. $\mu(y) = 1/y$

   C. $\mu(x) = x$

   D. $\mu(x) = 1/x$

   E. $\mu(x, y) = xy$

9. Initially the population of a certain species of fish in a lake is $1,000$. The intrinsic growth rate (in years) of the population is estimated to be 0.1 and the environmental carrying capacity to be $3,000$ fish. By the logistic model, two years later the fish population in the lake will be approximately [Hint: $y = y_o K/(y_o + (K - y_o)\mathrm{e}^{-rt})$]

   A. $1,832$

   B. 871

   C. $1,649$

   D. $1,137$

   E. 702

10. The general solution to the differential equation $y'' - y' = 0$ is

    A. $y(t) = C_1 + C_2\,t$

    B. $y(t) = C_1\,\mathrm{e}^t + C_2\,\mathrm{e}^{-t}$

    C. $y(t) = C_1 + C_2\,\mathrm{e}^{-t}$

    D. $y(t) = C_1 + C_2\,\mathrm{e}^t$

    E. $y(t) = C_1 t + C_2\,\mathrm{e}^t$

11. Let $y_1(t)$, $y_2(t)$ be two solutions to the equation $y'' - 2\,t\,y' + \sin(t)\,y = 0$. Then, by Abel's theorem, the Wronskian $W(y_1, y_2)(t)$ of the solutions $y_1(t)$, $y_2(t)$ is

    A. $C\mathrm{e}^{t^2}$

    B. C

    C. $C\mathrm{e}^{-t^2}$

    D. $C\mathrm{e}^{-2t}$

    E. $C\mathrm{e}^{2t}$

12. The characteristic polynomial of a constant coefficient linear, homogeneous differential equation is (r^2+1). Then the general solution to the equation is

    A. $c_1 \sin t + c_2 \cos t$

    B. $c_1 t \sin t + c_2 t \cos t$

    C. $c_1 e^{-t} + c_2 t e^{-t}$

    D. $e^{-t}(c_1 \sin t + c_2 \cos t)$

    E. $c_1 e^{t} + c_2 e^{-t}$

13. The function $y(t) = 1/t$ is a solution of $2t^2 y'' + 3ty' - y = 0$. Suppose you are using reduction of order to find another solution to the equation in the form $y_2(t) = v(t)y_1(t)$. Then $v(t)$ satisfies the differential equation

    A. $2tv'' - v' - v = 0$

    B. $2tv' - v = 0$

    C. $v'' + \frac{v'}{t} = 0$

    D. $2tv'' - v' = 0$

    E. $v' + \frac{v}{t} = 0$

14. Given that the exponential function (e.g., $y = e^{rt}$) is a possible solution to a *second order, homogeneous, constant coefficient, linear ordinary differential equation*, which of the follow is **NOT** a possible solution?

    A. Exponential multiplied by a linear function

    B. Exponential multiplied by a Sine or Cosine function

    C. Sine or Cosine function

    D. Sine or Cosine multiplied by a linear function

    E. Linear function