

MTH 256 – Sec 3.5 Sample Problems

6. Find the general solution of

$$y'' - 6y' + 9y = 0.$$

Sol. First write the characteristic equation:

$$r^2 - 6r + 9 = 0$$

which can be factored into

$$(r - 3)^2 = 0.$$

Thus the root $r = 3$ is a double root. One solution is $y_1 = e^{3t}$, but the other solution must be linearly independent: $y_2 = te^{3t}$. Therefore the general solution is

$$\boxed{y = c_1 e^{3t} + c_2 t e^{3t}}$$

12. Solve the initial value problem

$$y'' - 6y' + 9y = 0; \quad y(0) = 0, y'(0) = 2$$

Sol. From above we have that the general solution is

$$y = c_1 e^{3t} + c_2 t e^{3t}.$$

Applying the first initial condition $y(0) = 0$ gives $c_1 = 0$. To apply the second initial condition, we first need the derivative of the general solution

$$y' = 3c_1 e^{3t} + 3c_2 t e^{3t} + c_2 e^{3t}.$$

Substituting in $c_1 = 0$ and $y'(0) = 2$ gives $c_2 = 2$. Thus our solution is

$$\boxed{y = 2t e^{3t}}$$

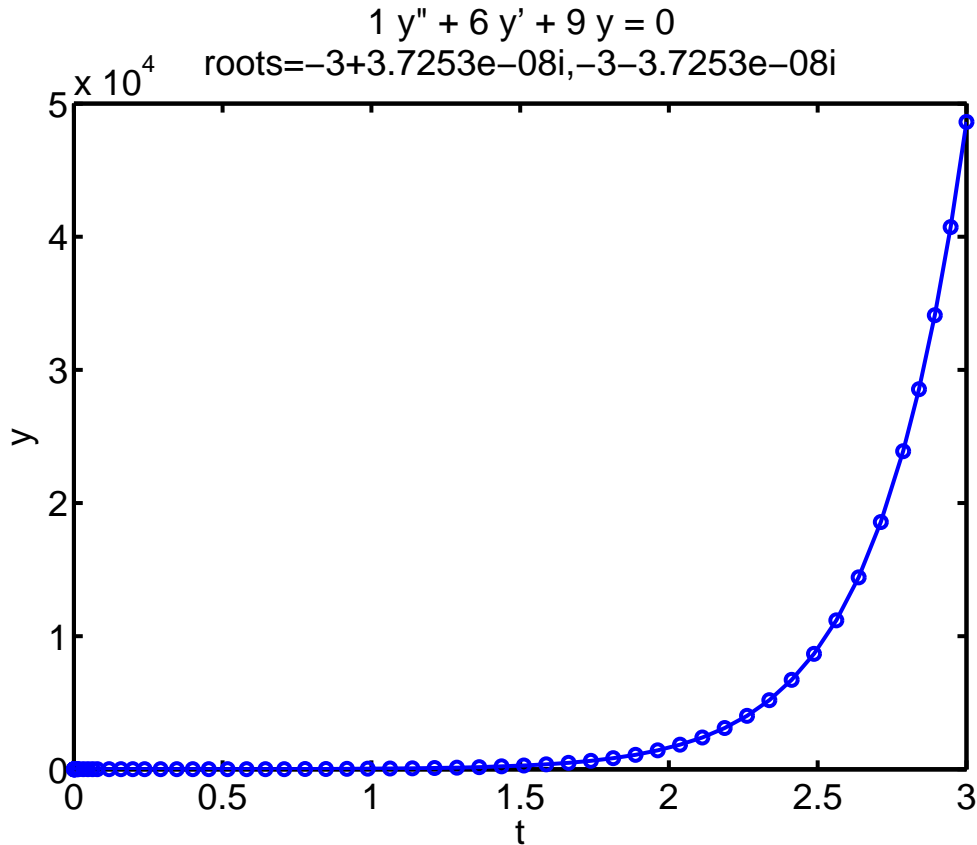
The plot of the solution using the code from the web with the command `cc2plot(1,6,9,0,2,0,3)` is shown in Figure 1.

4. Find the general solution of

$$4y'' + 12y' + 9y = 0.$$

Sol. First write the characteristic equation:

$$4r^2 + 12r + 9 = 0$$



which can be factored into

$$(2r + 3)^2 = 0.$$

Thus the root $r = -3/2$ is a double root. One solution is $y_1 = e^{-3t/2}$, but the other solution must be linearly independent: $y_2 = te^{-3t/2}$. Therefore the general solution is

$$y = c_1 e^{-3t/2} + c_2 t e^{-3t/2}.$$

15. Solve the initial value problem

$$y'' + 12y' + 9y = 0; \quad y(0) = 1, y'(0) = -4$$

Sol. From above we have that the general solution is

$$y = c_1 e^{-3t/2} + c_2 t e^{-3t/2}$$

Applying the first initial condition $y(0) = 1$ gives $c_1 = 1$. To apply the second initial condition, we first need the derivative of the general solution

$$y' = -\frac{3}{2}c_1 e^{-3t/2} - \frac{3}{2}c_2 t e^{-3t/2} + c_2 e^{-3t/2}.$$

Substituting in $c_1 = 1$ and $y'(0) = -4$ gives $c_2 = -5/2$. Thus our solution is

$$y = e^{-3t/2} - 5/2te^{-3t/2}$$

The plot of the solution using the code from the web with the command `cc2plot(4,12,9,1,-4,0,3)` is shown in Figure 2.

