

# On Finite Difference Methods for Non-Rectilinear Grids

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For the accurate solution of continuum mechanics problems, order of accuracy of difference equations is often less important than preserving physical symmetries. For example, if the divergence theorem is preserved under approximation, conservation of mass, momentum, and energy follows. A simple prescription for generating difference equations that preserve symmetries can be traced to early papers by Schulz and Goad. The prescription has three principal ingredients: (1) A symmetric, differentiable, compact test function  $S$  such that  $\mathbf{x} = \sum_v \mathbf{x}_v S(\mathbf{x} - \mathbf{x}_v)$ . (2) Exact integration of strain over control volumes. (3) The principle of virtual work. The difference operators and their adjoints on logically rectangular arrays of points  $\mathbf{x}_v$  yield the conservation relations of classical mechanics and the identities of vector calculus. The equivalence of the difference equations to variational forms provide estimates of order of accuracy. The considerable freedom in choosing the control volumes yields new results for adaptive mesh refinement and moving interfaces.

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