MMAM (MTH 420/520) LAB5: Fourier series and analysis. EXTRA

(**E=EXTRA**) Following the handout, the solution to $\frac{dx}{dt} + Kx = 0$ in R^N, with $x(0) = x_0$, can be constructed knowing the eigenvalues and eigenvectors of K. We can write the solution concisely using the "matrix exponential"

$$exp(-Kt) = V exp(-\Lambda t) V^{-1} = V \begin{bmatrix} exp(-\lambda_1 t) & & \\ & exp(-\lambda_2 t) & \\ \dots & & exp(-\lambda_n t) \end{bmatrix} V^{-1}. \tag{1}$$

Recall that the steps are

- Diagonalize $K = V\Lambda V^{-1}$ where Λ is the diagonal matrix of eigenvalues, and V is the matrix of eigenvectors. Change the variables to $w = V^{-1}x$ and derive the equation $\frac{dw}{dt} + \Lambda w = 0$
- Write out the initial condition x_0 in the new basis as $w_0 = V^{-1}x_0$
- Solve for w
- Transform back to get $x(t) = Vw(t)$
- (Identify the matrix exponential in the solution, $exp(Kt) = V exp(\Lambda t) V^{-1}$).

In summary, the solution is $x(t) = exp(-Kt)x_0$.

Follow the process above for the matrix $K = [2, -1; -1, 2]$, and $x_0 = [1, -1]^T$.