Linear Algebra Review Sheet for MTH 451/551

Instructions: *The concepts below are crucial for your understanding of the material in MTH 451-551. Most of them were covered in the prerequisite class MTH 341 and/or in the recommended class MTH 342. Please practice !!!*
You can come to Linear Algebra Clinic Mondays 5:00pm- in my office (weeks 2, 3, 4).

1. **Complex numbers**: arithmetics, conjugate, modulus, Euler formula

Find $\bar{z}, |z|, z_1+z_2, z_1 z_2$, for $z = 2-3i, z_1 = 5, z_2 = -4+\sqrt{2}i$, $Arg(z), Arg(z_1), Arg(z_2)$.

2. **Vectors:** linear independence, basis, linear span of vectors, dimension, change of basis; *orthogonal set of vectors; orthonormal set of vectors; Gram-Schmidt orthogonalization.*

Consider $u = [1, 2, 0]^T$, $v = [-1, 0, 5]^T$. Check if (u, v) are linearly independent, orthogonal, orthonormal. Find a third vector w if possible so that (u, v, w) is a basis for $\mathbb{R}^3$. Write $a = [2, 0, -5]^T$ in this basis. Can you find a vector $\tilde{v}$ so that $(u, \tilde{v})$ is an orthogonal (orthonormal) set (an orthogonal set spanning $Col(u, \tilde{v})$) ? Repeat for $(u, \tilde{v}, \tilde{w})$ for some $\tilde{w}$ that you would determine. Is it possible to do this for $(u, \tilde{v}, a, \tilde{w})$?

3. **Rectangular complex matrices:** $Col(A), Range(A), Ker(A), Null(A), A^T, A^*, A^*A$. Linear solvable systems, underdetermined and overdetermined systems.

Work with $A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, and $A = \begin{bmatrix} i & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $B = A^T$, $C = A^*$, and various submatrices and modifications of your choice of A, B, C.

4. **Square matrices:** singular, invertible; determinants; inverse matrix. *Orthogonal matrix, unitary matrix.*

Work with
$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$,
$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, A = \begin{bmatrix} i & 1 \\ -i & 1 \end{bmatrix}, A = \begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}. A = \begin{bmatrix} i & i \\ -i & i \end{bmatrix}, A = \begin{bmatrix} i & i \\ -i & 0 \end{bmatrix}$.

5. **Eigenvalue problem:** find eigenvalues and eigenvectors. Diagonalizable matrices. *Algebraic multiplicity and geometric multiplicity of an eigenvalue. Orthogonally (unitarily) diagonalizable matrices.*
Use examples as in 4.