MTH 451-551, Fall 2017, Assignment 2. Each problem is worth 5 points.

Instructions: *Please write neatly using proper mathematical notation, and show all the details necessary. As for code, you can show the snippets of the code, but please do not just print things from your MATLAB screen.*
Your paper should present what you have learned and concluded from the exercises, rather than a bunch of graphs stapled together.
You can discuss your problems with others, but all the solutions have to be written up individually. Please work independently on the computational part. 451 students can always solve the part for 551, if they so prefer.
Extra credit is turned in on separate paper.

1. Solve 4.5

2. See problem 4.4. (451) check if it is true (and if yes, prove) that if A and B are unitarily equivalent then they have the same singular values. (551) solve the entire problem 4.4.

3. (Computational) Follow this example which (i) loads an image, (ii) converts it to grayscale, (iii) constructs its svd, and then (iv) displays its lower rank approximation.

```
>> beaver = imread('Oregon_State_Beavers2_thumb.jpg');
>> graybeaver = rgb2gray(beaver);
>> imshow(graybeaver)
>> [U,S,V]=svd(double(graybeaver));
>> k=50;imshow(U(:,1:k)*S(1:k,1:k)*V(:,1:k)')
>> k=10;imshow(U(:,1:k)*S(1:k,1:k)*V(:,1:k)')
```

Now, use your OWN picture. (If it is high resolution, you may have to select just a part of it or use some image processing software to save it in lower resolution. More than 300×200 pixels might be too much for your local computer to handle.)

Construct lower rank approximation with $k = 5, 10, 50$ etc. Show to your best friend/-family the approximations. Do they recognize you? For which k yes and for which k no?

4. (Complexity) Calculate (by hand) the number of flops needed to compute the inner product $x' * y$ for vectors length n as in Problem 4 from Assignment 1. Do you believe that the timings you obtained agree with this calculation when you used your own loop, and when you used MATLAB?

Extra: repeat 3 in NUMPY/SCIPY.

Practice before exam/quiz. (Do not turn in).

(a) Come up with your own example of U, Σ, and V. Suggested choice of U and V are rotation matrices corresponding to some chosen angles. (Are these good choices?) Calculate $A = U\Sigma V^T \in \mathbb{R}^{2\times 2}$.

(b) Provide a drawing as suggested in Pbm 5.3 (b) and verify the details of that drawing.

(c) How do you choose U, V to create a symmetric matrix?

(d) What happens if $\sigma_2 = 0$?