MTH 451-551, Fall 2017, Assignment 4. Each problem is worth 5 points.

Instructions: *Please write neatly. Instructions for code etc as in Assignment 2.*
Extra credit is turned in on separate paper. (You can do 3-4 in python).

1. Suppose A is a non-singular $m \times m$ matrix with $\| A \|_2 = 10$, and $\| A \|_F = 11$. Give the sharpest possible bound on the condition number $\kappa(A) = \| A \|_2 \| A^{-1} \|_2$. **Hint:** Lecture 5 contains a theorem that will help you.
(451 students can assume $m = 2017$.)
Provide an example of such a matrix.

2. Find the condition number and the relative condition number for evaluation of $f(x) = x_1^2 + Ax_2^2$, where $x = [x_1, x_2]^T$. (451 solve this with $A = 1$, 551 use $A = 5$). **Extra credit:** consider any $A \geq 1$ or any $0 < A \leq 1$.

3. (Computational) Consider the polynomial (a) $p(x) = (x-3)^7$. Write it in the form (b) $p(x) = x^7 + a_6 x^6 + \ldots a_0$. Use MATLAB to plot $p(x)$ with $x \in [2.9, 3.1]$, and at least 100 points. Use the form a), and the form (b). Discuss your observations in the context of round-off error and stability of computer arithmetic.

4. (Computational). It is known that $e = \sum_{k=0}^{\infty} \frac{1}{k!}$.
a) Write a loop that will attempt to evaluate e by summing the finite number N of the terms of the series, from $k = 0$ to N. (Evaluate $k!$ by multiplication.) Compare the solution with the value of $e = exp(1)$ provided by MATLAB. Try several values of N.
b) Compare the result of a) with that when summing from $k = N$ to $k = 0$.
c) Now decide which N to use based on the value of the summand. N should be the smallest index k for which $\frac{1}{k!} < \varepsilon_{machine}$. Repeat a) and b).

(551) Discuss the above in view of Lecture 15 (see problem 15.1.e).