MTH 451-551, Fall 2017, Assignment 7. Each problem is worth 5 points.

In this assignment you will work with iterative methods for solving a linear system and for finding eigenvalues.

1. (451 and 551) Implement the conjugate gradient algorithm and steepest descent algorithm. Follow the example from class on conjugate gradient. Use $A = [1, 0; 0, 2]$, trivial initial guess, and the right hand side b corresponding to $x = [1; 1]$.
The code below can get you started (you must initialize properly and work in a loop.)

```
if cgflag == 0, p=r; else beta = -r'*A*p/(p'*A*p), p = r+beta*p; end
alpha=(r'*p)/ (p'*A*p), x = x + alpha*p, r = r - alpha*A*p,
```

How many iterations are needed before $\| r \| < 10^{-5}$?

2. (451 and 551). Do this calculation on paper (similar to Pbm 38.4). Consider the matrix A from Assignment 6, problem 2, (451 can assume with $N = 10$).
(a) Estimate the number of flops needed to solve the problem $Ax = b$ to the relative accuracy of 10^{-4} (as stated in (38.9)).
(b) (551) If Jacobi method is used, the eigenvalues of G_{JAC} for this matrix are given by $cos(j\pi h)$. Prove this.
(c) Compare (a) to the number of iterations and the number flops needed to solve this problem to the relative accuracy of 10^{-4} provided by the a-priori estimate derived in class for Jacobi method.
(d) (551) Compare to the number of flops needed by QR and by Cholesky to solve the same problem directly.

3. (451 and 551). Experiment with the power iteration (Algorithm 27.1 in textbook), to find an approximation to the eigenvalues to a given matrix A.
Write the code (show me), and test on matrix A from problem 1. If possible, find an initial guess for which the power iteration converges to $\lambda = 2$ (easy), and a guess for which it converges to $\lambda = 1$ (hard).

4. (451 and 551) Implement Rayleigh Quotient Iteration (Algorithm 27.3) and discuss how fast it converges to the eigenvalues of matrix A from Assignment 6, problem 2, with $N = 10$. Try at least three significantly different initial guesses. Identify the index of the eigenvalues they converge to.
551: Theorem 27.3 says the iteration converges cubically. Verify this on your examples.