MTH 452-552/Winter 2005, Assignment 1, due Friday, 1/18

Students registered for 452 solve 3 out of 4 (or all for extra credit). Students registered for 552 solve all 4 problems.

1. Consider the *logistic* equation i.e. $u' = f(t,u) = ku(a-u)$ with initial condition $u(0) = u_0$, and with $k = 1, a = 4$.
   i) Discuss the well-posedness of this IVP (verify continuity and L-continuity, on what region is it satisfied, is the existence and uniqueness result global or local, formulate what we know from theory).
   ii) Find the analytical solution using any applicable method.
   iii) Plot (in MATLAB) or sketch by hand the direction fields and discuss the behavior of solutions for various values of $u_0 = -1, 0, 3, 5$. (Some of these solution are called equilibrium, attracting, and repelling solutions).
   Extra: what are the applications ? what is the meaning of constants k,a ?

2. Consider the IVP $u'' + u = 0, u(0) = 1, u'(0) = 0$. Write it as a first order system. Solve it in both set-ups. What is the L-constant for this system ? (Consult Appendix A1-A3 for various matrix norms) . What are the eigenvalues of the matrix of this system ?
   Extra: what are the applications ? Plot (by hand) solution to the original ODE and to the system. Relate both formulations.

3. Solve 1.1 from text.

4. MATLAB: use the difference formulas $D_- f, D_0 f, \hat{D}_+ f$, where the latter is the one-sided second-order accurate formula, to approximate the derivative of $f(x) = \cos(x)$ at $x = .5$. Use h ranging from $1E-1$ down to the machine epsilon (step by a factor of 1/10). Compare the approximation with the exact value. Discuss behavior of the error (confirm theoretical order of convergence and reveal instability which occurs for very small h). (Use `loglog` plot).