MTH 452-552/Winter 2008, Assignment 2, due 2/1

452 students solve problems 1-3, 552 solve all problems.

1. Show that trapezoidal method is of second order for non-autonomous case.

2. Consider the θ-method defined as

$$U^{n+1} = U^n + h\left[\theta f(U^n) + (1-\theta) f(U^{n+1})\right]$$

   for an autonomous ODE, where $0 \le \theta \le 1$ is a parameter.
   Verify that the local truncation error of the method is at least first-order.
   For what value of θ is the method second-order ?.

3. (MATLAB) Consider the IVP
   $f(u,t) = \lambda u + sin(t)$, $y(0) = 1$ for $0 \le t \le 10$.
   i) Implement FE and BE methods for this problem.
   ii) Plot the exact solution and the approximate solutions obtained with FE, BE with $h = 0.1$ and $h = 0.2$, when $\lambda = -5$. Discuss the behavior of the error from the plot.
   iii) Find the global error for each h by taking $e_h := \max_n\{|U^n - u(t_n)|\}$. Consider $h = 0.1, 0.01, 0.001$. Does the error behave as predicted by theory ? Compare how fast/slow the algorithm runs for various values of h.
   Extra: implement trapezoidal and midpoint methods and repeat iii).

4. Solve 5.9(b,c) or 5.13