MTH 452-552 Assignment 1. **FirstName LastName**
Prof. M. Peszynska Due: 01/12/2018

Problem 1. Given a fixed parameter $\tau > 0$, consider the (rough) ODE

$$u' = f(t,u), \quad t \in [0,T], \quad u(0) = 0, \tag{1}$$

with $f(t,u) = 1$, for $j\tau \le t < (j+\frac{1}{2})\tau$, and $f(t,u) = -1$ for $(j+\frac{1}{2})\tau \le t < (j+1)\tau$. Here $j = 0, 1, \ldots J$, and J is given.
(a) Verify the conditions for the well-posedness for $T = \tau/4$. Does the theory apply?
(b) Repeat (a) for $T = J\tau$ and $J > 1$.
(c) Find the solution $u(t), t \in [0,T], T = \tau J$, and $J > 1$.
Since f is not smooth, you should think carefully how you would relax the notion of the solution to (1)*. Describe your notion. Did you contradict (b)?*
(d, MTH 552) Propose a method to regularize f by a smoother function f_ε. Discuss the properties of the corresponding u_ε.

Problem 2. Implement FE method for the problem above and test its convergence when $\tau = 1$. (Use the HW template provided at `http://math.oregonstate.edu/~mpesz/latex.html` to see how to assess the error). Provide appropriate plots and tables.
(a) Consider the case $T = \tau/4$ first.
(b) Consider $J = 5, T = J\tau$ next.
(c) If possible, determine h so that the error $e(h)|_{t=T}$ is less than 10^{-4}.
(d, MTH 552 students) Consider the regularized problem as in 1.(d), and apply FE to this problem. (You can use ode45 with a very small time step as a proxy for the exact solution to the regularized problem). Vary both ε and h. What choice yields the best results?

Extra project 1, to be submitted in CANVAS. Implement the linear pendulum problem in MATLAB with your own FE solver. You can also use `python`.
(a) Make plots in t to demonstrate the difference between the exact solution and your numerical solutions. Assess the error in the displacement (angle) θ and in the velocity $\dot\theta$.
(b) Prepare a demonstration of pendulum as in the movie on class website. The end of pendulum should have your initials, with an appropriate marker to delineate the exact vs approximate solution. Make a movie, and upload the movie to youtube. Provide the link to the youtube movie in the document on CANVAS.