

Problem 1. Given a fixed parameter $\tau > 0$, consider the (rough) ODE

$$(1) \quad u' = f(t, u), \quad t \in [0, T], \quad u(0) = 0,$$

with $f(t, u) = 1$, for $j\tau \leq t < (j + \frac{1}{2})\tau$, and $f(t, u) = -1$ for $(j + \frac{1}{2})\tau \leq t < (j + 1)\tau$. Here $j = 0, 1, \dots, J$, and J is given.

(a) Verify the conditions for the well-posedness for $T = \tau/4$. Does the theory apply?

(b) Repeat (a) for $T = J\tau$ and $J > 1$.

(c) Find the solution $u(t), t \in [0, T], T = \tau J$, and $J > 1$.

Since f is not smooth, you should think carefully how you would relax the notion of the solution to (1). Describe your notion. Did you contradict (b)?

(d, MTH 552) Propose a method to regularize f by a smoother function f_ε . Discuss the properties of the corresponding u_ε .

Problem 2. Implement FE method for the problem above and test its convergence when $\tau = 1$. (Use the HW template provided at <http://math.oregonstate.edu/~mpesz/latex.html> to see how to assess the error). Provide appropriate plots and tables.

(a) Consider the case $T = \tau/4$ first.

(b) Consider $J = 5, T = J\tau$ next.

(c) If possible, determine h so that the error $e(h)|_{t=T}$ is less than 10^{-4} .

(d, MTH 552 students) Consider the regularized problem as in 1.(d), and apply FE to this problem. (You can use ode45 with a very small time step as a proxy for the exact solution to the regularized problem). Vary both ε and h . What choice yields the best results?

Extra project 1, to be submitted in CANVAS. Implement the linear pendulum problem in MATLAB with your own FE solver. You can also use `python`.

(a) Make plots in t to demonstrate the difference between the exact solution and your numerical solutions. Assess the error in the displacement (angle) θ and in the velocity $\dot{\theta}$.

(b) Prepare a demonstration of pendulum as in the movie on class website. The end of pendulum should have your initials, with an appropriate marker to delineate the exact vs approximate solution. Make a movie, and upload the movie to youtube. Provide the link to the youtube movie in the document on CANVAS.