MTH 452-552 Assignment 2. **FirstName LastName**
Prof. M. Peszynska Due: 01/26/2018

Problem 1, theoretical. (a) If possible, find a consistent scheme $D_h^{exotic}u(t)$ based on the values $u(t+2h)$, and $u(t-h)$. What is the order of LTE? Next, try to find a third order scheme by including the point $u(t)$.
(b) Confirm that the order of LTE for Heun method and for trapezoidal methods is $O(h^2)$. What assumptions on the smoothness of the exact solution $u(t)$ are you making? (452 students can assume that the problem is autonomous).

Problem 2, computational. Implement Heun method and trapezoidal methods, and test their accuracy. Test the problem on each of the three examples below, choosing T and the largest h wisely. Comment on the cost of each method.

(1) $$u'(t) = -tu,\ u(0) = 1,$$

(2) $$u'(t) = -tu^2,\ u(0) = 1,$$

(3) $$u'(t) = f_{\text{HW1,Problem1}},\ u(0) = 0; T > 2.$$

If the scheme is implicit and the problem is nonlinear, use fixed point iteration. (Recall the information about implicit schemes in the Interlude part of class notes).

Extra project 2, to be submitted in CANVAS. `The solution should comprise the code in MATLAB/other, as well as graphs, all put together in one PDF file`.
Prepare a demonstration of FE, BE, Heun and trapezoidal methods for (1) as follows.
(a) Plot the direction fields, the exact solution, and the numerical solutions corresponding to each of the schemes over the interval $(0, h)$.
(b) Show how the slopes are computed explicitly in the FE and Heun methods, and implicitly in BE and trapezoidal schemes, and how we advance the solution from $u(0) = U^0$ to $U^1 \approx u(h)$ by drawing the lines of appropriate slopes starting from the point $(0, u(0))$.
(c) Repeat for (2)–(3).