Problem 1, theoretical. [**452 students do one, 552 students do both.**] (a) Two students are discussing LTE and implementation of a first order scheme for $u' = f(t, u)$. John thinks that the scheme

$$(1) \qquad\qquad U^{n+1} = U^n + hf(t_{n+1}, U^n)$$

is easy to implement and should be consistent, even though he recognizes the scheme is neither BE nor FE. Mary says the scheme seems like an unfinished first step of BE, and is concerned about its consistency. What do you think? Is either of them right?

(b) Anusha and Yi take the discussion in a different direction, and consider a system

$$(2) \qquad\qquad u' = f(u), \quad v' = g(u, v).$$

Anusha proposes the so-called *sequential method*

$$(3) \qquad\qquad U^{n+1} = U^n + hf(U^n), \quad V^{n+1} = V^n + hg(U^{n+1}, V^n),$$

but Yi is concerned about consistency, and would prefer a fully implicit scheme. They argue about the ease of implementation and accuracy. What do you think?

Problem 2, theoretical. (a) Carry out the calculations to derive the 2-step Adams-Bashforth scheme.

(b) Consider a two-stage RK method in which $a_{11} = 1, a_{22} = 1, c_1 = 0, c_2 = \frac{1}{5}$. If possible, find the remaining coefficients a_{21}, a_{12}, b_1, b_2 so that the scheme is consistent (to the highest possible order).

No extra project 3 this time. Save your energy for the Midterm!