MTH 452-552 Assignment 4. **FirstName LastName**
Prof. M. Peszynska Due: 02/09/2018

Problem 1, computational. Warm-up: reproduce example 6.2 from textbook. (do not turn in). Now, consider Ex.4.4 and 4.5 from class notes. For these, produce a table similar to that in example 6.2 from textbook.

Problem 2, computational. Consider the system of equations $x' = y, y' = -x$ with initial conditions $x(0) = 1, y(0) = 0$. Exact solution to this system can be found easily by converting it to an IVP for second order ODE.

(a) Implement solution to this system using FE, BE, and trapezoidal method. Plot the solution obtained for $t \in [0, T], T = 2\pi$ in the phase plane. Use $h = 0.1$. Discuss the qualitative properties of the FE, BE, and trapezoidal method solutions with respect to the exact solution. Which one agrees better with the true solution?

The code below may be useful.

```
%% before the loop
A=[0,1;-1,0]; uFE(1)=1;uFE(2)=0;
....
%% inside time loop for FE
 uFE = uFE + h*A*uFE;
 ...
%% inside time loop for BE
 uBE = (eye(2,2)-h*A)\uBE;
 ...
%% plotting uBE in the (x,y) plane [phase plane]
%% at each time step
hold on; scatter(uBE(1),uBE(2));hold off;
```

(b) Plot the norm of the error $\| u(t_n) - U^n \|_\infty$ calculated at every time step in function of t_n. Here $u(t) = [x(t), y(t)]^T$. Does the error increase with n?

(c) Instead of the error in (b), one may be interested in the distance of (X^N, Y^N) from the origin. (Here N is the last time step.) Report on this quantity for each method. (One can prove something about $x(t)^2 + y(t)^2$ using implicit differentiation).

(d) (Extra) What happens if you use $h = 1$?

(e) (Extra Extra) Add to your comparisons the use of MATLAB's code `ode45`.

Extra project 4: Lorenz system is the well known example of chaotic behavior. Here we use a particular set of coefficients to demonstrate the sensitive nature of numerical computations more pronounced than in Pbm.2

$$u_1'(t) = 10(u_2 - u_1), \tag{1}$$

$$u_2'(t) = u_1(28 - u_3) - u_2, \tag{2}$$

$$u_3'(t) = u_1 u_2 - \frac{8}{3} u_3. \tag{3}$$

Implement FE, and some other methods (BE, Heun, trapezoidal, `ode45`, ...) for the problem. Use initial conditions $[0; 2; 10]^T$. and $T = 10$. The famous loops are created when plotting the u_1, u_2 together in the phase plane (you can also plot in 3D). Compare the dynamics for different methods, different h, and for somewhat altered initial conditions.