MTH 452-552 Assignment 5-6 (double credit). **FirstName LastName**
Prof. M. Peszynska Due: 02/23/2018

Problem 0, extra credit. Use your own words to come up with not-so-serious real-life analogy to "consistency", "zero-stability", "absolute stability", "A-stability", and "L-stability".

Problem 1, theoretical; routine. Consider a generic two-step LMM $U^{n+2} + \alpha_1 U^{n+1} + \alpha_2 U^n = h(\sum_{j=0}^{2} \beta_j f(U^{n+j})$. (452 solve (a) or (b)). 552 consider the general case of r-step method for (a) or do both (a) and (b) for a two-step method.
(a) Prove that it is necessary for consistency that $\rho(1) = 0$. (Hint: consider $u' = 0$).
(b) Prove also it is necessary for consistency to have $\rho'(1) = \sigma(1)$. (Hint: use $u' = \lambda u$).
In your calculations you should use Taylor expansions and the LTE.
(Extra:) Are these conditions sufficient for the autonomous case $u' = f(u)$?

Problem 2, theoretical; non-standard. If possible, design a consistent four-step method which is not zero-stable. Assume $\xi = 0$ is a double root of $\rho(\xi)$.
(Extra) If possible, design an inconsistent one-step method which is not zero-stable and one which is not zero-stable.

Problem 3, computational, non-standard schemes. Plot the stability region $\mathcal{R}^{\mathcal{ABS}}$ for the scheme in Problem 2, and the scheme associated with D_h^{exotic} from Homework 2. Discuss what you see and how you would find the time step h when $\lambda = -100$.

Problem 4, computational, practical. (a) Plot the stability region for the Heun method and Improved Euler methods. (Calculate $R(z)$ and plot contours).
(b) Consider the three-step AB, AM, and BDF schemes. (Sections 5.9 and 8.4 of the textbook). (Set-up $\rho(\cdot), \sigma(\cdot)$ and plot their ratio for $e^{i\theta}$).
Find h (if possible) so each of these methods is stable when $\lambda = -10+i$, and when $\lambda = 1+10i$. Mark these points on the stability plot for each method. (Please use the same scale for all three methods). Discuss which method seems preferable.
(Extra:) plot the order stars for these methods. (See Section 7).

Extra project 5: not this time (plenty of work), but next!