MTH 452-552 Assignment 7. **FirstName LastName**
Prof. M. Peszynska Due: 03/02/2018

Consider a simple ODE which is easy to solve

$$u' = -10u, u(0) = 1; \qquad t \in [T0, T], \tag{1}$$

and a kinetic ODE, with application to adsorption phenomena

$$u' = -10(u - p(t)), u(0) = 1; \qquad t \in [T0, T], \;\; p(t) = \frac{t}{1+2t}. \tag{2}$$

When choosing time steps for accuracy reasons, it is OK to consider "nice" h: $T/h \in \mathbb{N}$.

Problem 0, warm-up (do not turn in). Plot the solution to (1). Use `ode45` to study the solution to (2) or (1). For example, see

```
f=@(t,u)(-10*u);[te,ue]=ode45(f,linspace(T0,T,1000),y0);
plot(te,ue,'r*',te,exp(-10*te),'k-');
```

Problem 1, theoretical. Confirm whether the Improved Euler method can be used along with FE in the embedded RK framework for truncation error estimates.

Problem 2, computational. [552 do (a-d), 452 do three of (a-d).] `Code templates are available in class notes.`

(a) Implement the pair of FE+Heun methods for (1) to predict the truncation error a-posteriori in the first two steps of FE on $t \in [0, 1]$. (Compare with the true value $hu''(t)/2$). By trial and error find the time step h for which the LTE ≤ 0.1. Compare with the stability restriction.

What about when $t \in [1, 2]$? (modify the initial condition to $u(1) = e^{-10}$).

(b) Implement the Richardson strategy of time step doubling to estimate a-posteriori the global error for (1) at $T = 1$ ($T0 = 0$); compare with the true error. Test with $h = 0.05$ and smaller.

(c) Repeat (a) for (2). **Hint:** the exact solution of the problem may not be easy to compute. You can use the `ode45` solution calculated on a very fine grid as a proxy. For example,

```
...
[te,ue]=ode45(f,linspace(T0,T,10000),y0);
exact = @(t)(interp1(te,ue,t));
...
```

(d) Repeat (b) for (2).

Extra project 7: (a) Implement full variable/adaptive time stepping strategy for (2) similar to Pbm 2a. Test on scalar examples from class notes. (In each time step predict LTE and decrease or increase time step. Make sure not to violate stability restrictions).

(b) Define a global error estimation strategy for the trapezoidal method similar to Pbm 2b. Test it on (2).