

MTH 453-553/Peszynska/Spring 2008

Assignment 4 (due on Final Exam)

453 do 1, 3, one of 2i, 2ii. **553** do 1-3.

1. Sketch the characteristics and the solution at $t = 1$ to i) $u_t + u_x = 0$ and ii) $u_t + uu_x = 0$ with the initial data $u_0(x) = 1$ if $|x| \leq 1$ and $u_0(x) = 0$ otherwise.
2. Consider $u_t + u_x - DU_{xx} = 0$. Analyze stability of i) the explicit (FB) and of ii) fully implicit (BB) upwind scheme. (diffusion is treated by the usual second order difference operator). In i) all spatial derivatives are explicit in time, in ii) all are implicit.
Extra: can anything be gained if one of spatial derivatives is taken explicitly, and the other implicitly in time ?
3. (MATLAB) Plot amplification factor(s) derived for FC, FB, BB. for advection-diffusion equation. (You can plot them in the complex plane and plot their magnitude in function of the wave number θ). Choose different $D, h, \Delta t$ so that different stability/instability regions can be seen.
4. **Extra:** implement FB scheme for advection-diffusion equation on $(0, 1)$, with initial and boundary data as in class; use outflow condition at $x = 1$. Test with different values of Péclet number and different $h, \Delta t$.