MTH 453-553 Assignment 2. **FirstName LastName**
Prof. M. Peszynska Due: 05/09/2018

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This time I don't provide complete code, because you can glue it from different
pieces we discussed thus far.  Please attach a full listing of your code for exactly
one of the schemes you use.
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Problem 1: stability and convergence of FE, BE, and CN. (453 implement FE and at least one other, 553 implement all three). Implement the schemes for the heat equation with homogeneous Dirichlet BC, and f and $u_0(x)$ found so that $u(x,t) = e^{-\pi^2 t}\sin(\pi x)$.

$$u_t - u_{xx} = f, \;\; x \in (0,1), \; 0 < t \leq T, \;\; u(0) = 0 = u(1). \tag{1}$$

In testing, use some T for which $0.2 \leq \max_x u(x,T) \leq 0.3$. Consider $M = 10, 20, 50, 100$. For each M, you must make a choice of Δt. If possible, choose Δt for which the scheme (i) is unstable, and (ii) another Δt for the scheme is stable, but not optimally convergent, (iii) another Δt for which the scheme is optimally convergent, and (iv) another for which further decrease of Δt does not change the error.
Be concise: use theoretical considerations along with the numerical experiments to make your point. Write a careful report with well chosen supporting evidence, rather than a core dump. Points will be taken off for any work that is not to the point.

Problem 2. Consider the homogeneous heat equation

$$u_t - (ku_x)_x = 0, \;\; x \in (0,1), \; 0 < t \leq T u(0) = 2, u(1) = 1, \; u(x,0) = 1. \tag{2}$$

with a piecewise constant coefficient $k(x) = \begin{cases} k_1, x < 0.5 \\ k_2, x > 0.5 \end{cases}$. Let $k_1 = 1, k_2 = 10$.

Find the stationary solution by hand calculation (see pbm 3 in HW 1). Denote it as $u_\infty(x) = \lim_{t\to\infty} u(x,t)$.

Now implement a fully discrete scheme of your choice for (2), and use $M = 10, 20, 50, 100$. Use time stepping $\Delta t = \Delta t(M)$ of your choice for which the scheme has optimal convergence (justify your choice). Report on the time $t_\infty(M, \Delta t)$, the smallest time t, for which

$$|\, u_\infty(0.5) - U_J^{n_\infty} \,| \leq 0.01. \tag{3}$$

Here J denotes the index of the spatial grid point corresponding to $x = 0.5$, and n_∞ corresponds to the time step corresponding to t_∞.
Extra credit: check convergence of your scheme using some fine grid solution. Do you expect optimal order?
