MTH 453-553 Assignment 5. **FirstName LastName**
Prof. M. Peszynska Due: 06/04/2018

`Show the code you develop for schemes other than upwind.`

Consider the IVP for advection-diffusion equation, with $a > 0, D > 0$, and $r \in \mathbb{R}$.

$$u_t + au_x - Du_{xx} + ru = f(x), \;\; u(x,0) = u_0(x), \;\; x \in (0,1), \; t \in (0,T). \tag{1}$$

Problem 1 (theoretical), double credit. Compare the advantages and disadvantages of three schemes for (1), each involving an upwind scheme applied to au_x and the usual handling of $-Du_{xx}$. **Choice of** r: MTH 453 students can assume $r = 0$. MTH 553 students consider $r = 0.01$. Consider any $r \neq 0$ for extra credit.

The schemes to consider are (i) fully implicit in time, (ii) fully explicit in time, (iii) implicit in diffusion and reaction and explicit in advection.

A. State each scheme.

B. For each scheme, discuss **computational effort** and **accuracy**. This discussion should be based on the facts derived in class or in the textbook; if you wish, you can include additional analyses. (You can assume periodic boundary conditions so that stability analysis via von-Neumann applies.)

B1. To help guide the answer on **computational complexity**, set $h = 0.1$, $D = 0.01$, and choose Δt depending on stability restrictions, and to minimize the error. Include an estimate of how many time steps are needed to solve the problem when $T = 1$.

B2. To help guide the answer on **accuracy**, assume that the error

$$\max_n \| (u(x_j, t_n) - U_j^n)_j \|_{\Delta,2} = 1. \tag{2}$$

What would be the error if you used $h = 0.01$ (and changed Δt appropriately)?

C. Provide a summary statement comparing the three schemes. (You can hypothesize how your answers would change, e.g., if $D \uparrow, r \uparrow$.)

Extra credit: implement at least one of the schemes in Problem 1, for an initial condition as in Assignment 4 for problem (1)a (smooth Gaussian initial data), and $f \equiv 0$. Show snapshots of the solution to demonstrate the coupled effects of advection, diffusion, and reaction.
