HW 1, MTH 482-582_W15/Peszynska

Please complete the following review problems on partial derivatives

(1) Consider the following transformation $(\xi, \eta) = T(x, y)$ with $\xi = x + y, \quad \eta = x - y$,
(a) Find DT. Check if DT is singular for any x, y.
(b) For some $u = u(\xi(x, y), \eta(x, y))$, express u_x in terms of u_ξ, u_η.
(c) Sketch $T([0, 1]^2)$.
(d) Express u_{xx} in terms of the first and second order partial derivatives of u with respect to ξ, η.
(e) Consider a particular example where

$$u(\xi, \eta) = sin(\xi) + exp(\eta).$$

Check your answers in (b) and (d).
(f) Work out the details of the inverse transformation T^{-1} and calculate its Jacobian DT^{-1} directly and from the inverse function theorem.

(2) Consider the following transformation $(\xi, \eta) = T(x, y)$ with $\xi = Ax + By, \quad \eta = Cx + Dy$, where A, B, C, D are real constants.
(a-c) Complete (a-c) from Pbm (1).
In (c), you can assume for simplicity that all A, B, C, D are positive.

(3) Consider the following transformation $(\xi, \eta) = T(x, y)$ with $\xi = x^2 + 3y, \quad \eta = x + 2y^2$,
(a-b) Complete (a-b) from Pbm (1).
(c) Sketch $T([0, \frac{1}{4}]^2)$.
(d) Extra: Sketch $T([0, 1]^2)$. Interpret in view of (b).

(4) Consider polar coordinates and $x = x(r, \theta), y = y(r, \theta)$ (equivalently, $r = r(x, y), \theta = theta(x, y)$.
Work out details to show that

$$u_{xx} + u_{yy} = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}.$$

`I know this calculation can be looked up` *`anywhere`*`. But please do it anyway.`