MTH 621/Peszynska/Fall 2008

Assignment 1

Please show all your work. Use proper mathematical notation. Plots can be approximate.

1. Let v be a constant.
   1) Find general solution to $u_t + vu_x = 0$.
   2) Sketch characteristics: consider $v = 0, v = 2, v = -1/2$ separately.
   3) Plot the solution in (x, u) plane for $t = 1, 10$ and for an initial condition $u(x, 0) = \frac{1}{1+x^2}$.

2. Find and plot characteristics for $(x^2 + 1)u_x + u_y = 0$. Suggest a curve along which an auxiliary condition can be given so that a solution can be found in some region $D \subseteq \mathbb{R}^2$. Also, suggest one along which such a condition will not give us a solution on any region.

3. Solve the equation $u_x + u_y = 1$ with the condition a) $u(x, 0) = 5$ and b) $u(0, y) = max(0, 1 - y^2)$, if possible. What behavior of solutions do you expect ? answer before and after you found the solution.