MTH 621/Peszynska/Fall 2008
Assignment 2
Please show all your work. Use proper mathematical notation.

1. Discuss existence and uniqueness of solutions to the following BVP:

$$u'' = x, x \in (0,1),$$

with homogeneous a) Dirichlet and b) Neumann boundary conditions.
Does the answer change when you consider the problem posed on $(-1,1)$ instead on $(0,1)$?

2. Let u solve $u'' + ku = 0$, where $k > 0$. Consider an IVP for this equation with $u(0) = 0, u'(0) = 1$ given.
Consider a BVP for this equation with $u(0) = 0, u(1) = 0$.
Discuss the well-posedness for both cases.

3. Find and sketch the regions in the xy plane where the equation

$$(1+x)u_{xx} + 2xyu_{xy} - y^2 u_y y = 0$$

is elliptic, hyperbolic, or parabolic.

4. Let k be an arbitrary real constant. Depending on k, determine the type and transform the equation $u_{xx} + 2u_{xy} + ku_{yy} = 0$ where k to a canonical form, by changing variable. Propose the general solution whenever possible.