MTH 621/Peszynska/Fall 2008
Assignment 3
(Do not turn in, use for practice on wave equation before midterm).

Consider

$$u_{tt} - c^2 u_{xx} = 0, \; x \in \mathbb{R}, t \in \mathbb{R} \tag{1}$$

$$u_{tt} - c^2 u_{xx} = f(x,t), \; x \in \mathbb{R}, t \in \mathbb{R} \tag{2}$$

$$u_{tt} - c^2 u_{xx} = 0, \; x \in (0,L), t \in \mathbb{R} \tag{3}$$

1. Prove the following property of convolution and give the assumptions under which it holds

$$\frac{d}{dt}(f * g) = f(0)g(t) + f' * g \tag{4}$$

2. Carry out calculations which show that $u(x,t)$ as given in class solves (2) with homogeneous initial conditions. Also, find the solution when initial conditions are inhomogeneous.

3. Compute the solution to (2) when $f(x,t)$ is given as $1, xt, e^{-x}, cos(x)$.

4. Use the formula from class for solution to (2) to show how that solutions depends on the data $f(x,t)$.

5. Use the formal solution we derived for (3) to show that it can be written in the form $h(x+ct) + h(x-ct)$.
Hint: Use the formula for $sin(A+B) - sin(A-B)$.

6. Derive formally the solution to (3) with i) homogeneous Neumman conditions, ii) with mixed homogeneous conditions.

7. Use the solution to (3) with special initial data $u(x,0) = sin(x), u_t(x,0) = 0$ with $L = \pi$ that we derived in class and compute the energy of that solution. Show that it remains constant.