MTH 621/Peszynska/Fall 2008
Assignment 4

1. Find the best approximation of $f(x) = x^2$ in $L^2(-1, 1)$ in the subspace spanned by $\{1, x\}$. What happens if we choose $L^2(0, 1)$ instead ?

2. Show that the functions $\{sin((n + \frac{1}{2})x)\}$ form an orthogonal set on the interval $(0, \pi)$. How about on $(-\pi, \pi)$? On $(0, \pi/2)$? Propose a change to make this set orthonormal, if possible. Can this set be used as a basis for all of $L^2(-\pi, \pi)$? (Hint: consider value of the functions at 0.)

3. Consider the Fourier series for the function $f(x) = x$ on $(0, \pi)$ extended to $(-\pi, \pi)$ in an i) even, ii) odd way, iii) by translation. What do we know about the way the Fourier series converges to $f(x)$ on $(-\pi, \pi)$? (Answer without computing the coefficients).
   Extra: Actually calculate the Fourier series in each case and plot the sum of the first few terms.