MTH 621/Peszynska/Fall 2008
Assignment 5

1. Use separation of variables to (formally) solve the damped wave equation $u_{tt} + ru_t = c^2 u_{xx}$ on $(-\pi, \pi)$, with homogeneus Dirichlet boundary conditions and with initial conditions same as for the wave equation. Consider $c > 0$ and a small damping parameter r $0 < r < 2c$.
   Extra: what happens if r is negative or large positive ?

2. Find the eigenvalues, eigenfunctions, and solve the diffusion equation with periodic boundary conditions on $(0, L)$.

3. Solve the advection-diffusion equation $u_t - ku_{xx} + vu_x = 0$ on $x \in \mathbb{R}, t > 0$, where $k > 0$ and v is a given constant velocity. **Hint:** change variable to $y = x - vt$. Use initial condition $u(x, 0) = box(x)$.
   Plot the solutions for $k = 1, 10^{-2}, 10^2$ and $v = 1, 10^{-2}, 10^2$ for a fixed time and discuss dependence of the solution on the Péclet number $\frac{v}{k}$ (ratio of advection to diffusion).