MTH 621/Peszynska/Fall 2015, Assignment 1

Please show all your work. Use proper mathematical notation.

1. [ **pts**] Let $v$ be a constant.
   1) Find general solution to $u_t + vu_x = 0$.
   2) Sketch characteristics: consider $v = 0, v = -1, v = 1/2$.
   3) Sketch the solution in $(x, u)$ plane for the initial condition $u(x, 0) = \frac{1}{1+x^2}$ for $t = 1, 2, 5$.

2. [ **pts**]Find and sketch characteristics for $(x^2 + 1)u_x + u_y = 0$. Suggest an auxiliary condition so that a solution can be found in some region $D \subseteq \mathbb{R}^2$. Also, suggest a condition for which a solution cannot be found.

3. [ **pts**] Solve the equation $u_x + u_y = 1$ with the condition a) $u(x, 0) = 5$ and b) $u(0, y) = max(0, 1 - y^2)$, if possible. If not possible, explain why. What behavior of solutions do you expect ? (Answer before and after you found the solution.)