MTH 622/Peszynska/Winter 2009
Assignment 1
Please show all your work. Use proper mathematical notation.

1. Verify whether the following functionals on $C^1([0,1])$ is linear i) $J(y) = \int_0^1 yy'dx$, ii) $J(y) = \int_0^1 \int_0^1 K(x,t)y(x)y(t)dxdt$, iii) $J(y) = \int_0^1 ysin(x)dx$, iv) $J(y) = y'(1/2) + y(0)$. v) $J(y) = f(y(0))$, where f is a given function.

2. Derive Euler-Lagrange equation for the functional $J(y) = \int_0^1((y')^2 - fy)dx$, where f is a given function.

3. Derive the PDE (finish class example) satisfied by the minimal surface problem (soap bubble suspended from a wire of a given shape). Determine the type of this second-order PDE.

4. If possible, find the stationary point (extremal) if the functional $J(y) = \int_0^1(y^2 + x^2y')dx,\ y(0) = 0, y(1) = \alpha$. (Your answer will depend on the value of α).