MTH 622/Peszynska/Winter 2009
Assignment 2
Please show all your work. Use proper mathematical notation.

1. Find distributional derivatives of order $1, 2, \ldots$ of $u(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$.
   Identify which of these derivatives (if any) are classical, weak.

2. Consider $Lu := xu'$ where u' denotes either classical (if it exists), or distributional derivative. If possible, i) find the classical solution $u \in C^1(\mathbb{R})$ to $Lu = 0$, ii) find the formal adjoint of L, iii) show that $u = cH(x) + c_1$ is a weak solution to $Lu = 0$.

3. (EXTRA) Show that, in the sense of distributions, $x\delta' = -\delta$.

4. Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Show that i) A is spd. ii) Show that $(x, y)_A := y^T Ax$ is an inner product. iii) Given a vector $f \in \mathbb{R}^2$, find the vector that minimizes the functional (also called quadratic form) $J(u) = \frac{1}{2}u^T Au - u^T f,\ u \in \mathbb{R}^2$. (Hint: find $J(u + \epsilon v)$ and proceed similarly as we did in order to find E-L equations, except here this time you minimize over $\mathbb{R}^2$ and not over some space of functions.

5. Verify that the bump function is a test function.