MTH 622/Peszynska/Winter 2016, Assignment 1. Part B

Please show enough work to justify your answer.
Use proper mathematical notation.

(1) Verify that $u(x,t) = -2xt - x^2$ solves the homogeneous diffusion equation with variable coefficients $u_t - xu_{xx} = 0$. Find its maximum on $[-2,2] \times [0,1]$ and verify if the maximum principle applies (it does not). Where does the proof of maximum principle break down for this equation ?

(2) Prove the comparison principle for the solutions of the diffusion equation $u_t - u_{xx} = 0$, with some initial and boundary conditions given as in text.
Assume that u, v are both solutions, and that $u \leq v$ on the part

$$[0,L] \times 0 \cup 0 \times [0,T) \cup L \times [0,T)$$

of the boundary of the region $Q_T = (0,L) \times (0,T)$. Show that $u \leq v$ on Q_T.

(3) Solve the "steady-part" of 5.2.7 (without using the explicit formula derived in part A).