MTH 622/Peszynska/Winter 2016, Assignment 2. Part B

Please show enough work to justify your answer.
Use proper mathematical notation.

(1) Solve the diffusion equation $u_t - au_{xx} + cu = 0$ on $\mathbb{R}$, with $u(x,0) = f(x)$. To handle the constant dissipation term, change variable $u(x,t) = e^{-ct}v(x,t)$.

(2) Solve the advection-diffusion equation $u_t - au_{xx} + cu_x = 0$ on $\mathbb{R}$, with $u(x,0) = f(x)$. To handle the advection term, change variable $y = x - ct$.

(3) Prove uniqueness of the diffusion equation $u_t - au_{xx} = F(x,t)$ on $\mathbb{R}$, with $u(x,0) = f(x)$ by the energy method. What assumptions do you need?

(4) **Extra:** Let u solve $u_t = \frac{1}{2}u_{xx}$. Show that $v(x,t) = \frac{1}{\sqrt{t}}e^{x^2/2t}u(x/t, 1/t)$ satisfies the backward heat equation $v_t = -\frac{1}{2}v_{xx}$.

(5) **Extra:** Provide the formal solution to (1), (2) using Fourier transform.