MTH 655, Winter 2011, Assignment 3 problems

1. Provide details for showing 3.2.8 is true. Hint: when estimating $|f(x) - I_\triangle f(x)|$ on some (a, b), get the expression to the form involving a convex combination of $f(x) - f(a)$ and of $f(x) - f(b)$. Then use the fact that if $\lambda \in (0, 1)$, then $|c\lambda + (1 - \lambda)d| \leq max(|c|, |d|)$, for any $c, d \in \mathbb{R}$.

2. Show details leading to 3.2.12.

3. Convince yourself of the sharpness of interpolation estimates and confirm them numerically. Take $u \in H^2 \setminus C^2$ and its interpolant $I_\triangle u$. Then determine experimentally the order of convergence of $\| u - I_\triangle u \|_W$ where $W = C^0$ and $W = L^2$. (Do this first for a smooth function in C^2 so you know for sure your estimates work). Consider only the "unfavorable" partitions.

4. Work out the details of the estimate for the quasi-interpolator, with $u \in H^1$, for $\| u - \tilde{I}_\triangle u \|_W$. Let $\Omega_h = [x_0 - 2h, x_0 + 2h]$ for some x_0 and some $h > 0$. Show

(B) $|u - \tilde{I}_\triangle u||_{x_0} \leq C\sqrt{h} \| u \|_{H^1(\Omega_h)}$

(A) $\| u - \tilde{I}_\triangle u \|_{H^m(\Omega_h)} \leq Ch^{1-m} \| u \|_{H^1(\Omega_h)}$, $m = 0, 1$.

Hint: To get started, write out the definition of the quasi-interpolant. Then to get (B), expand $u \in H^1$ as $u(x) = u(x_0) + \int_{x_0}^{x} u'(s)ds$, and estimate the integral using Cauchy-Schwarz inequality. Take a note of how you could estimate that integral if you had higher regularity than that $u \in H^1$. (A) can be obtained similarly.

5. Challenge: confirm the order(s) in 4 numerically. Note that you have to pick a function u for which no superconvergence can occur, i.e., one whose regularity is exactly that of H^1.