MTH 655, Winter 2017, Assignment 3, theoretical part

In this HW, in the theoretical part, you practice working with bilinear form and error estimates.

Solve 1. If 1 is too easy, solve 2, or both for extra credit. Problem 3 is for extra credit only.

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1. Bilinear form and variational form (A) Consider the form

$$a(u,v) = \int_{-1}^{1} 3u'v' + \int_{-1}^{1} (2x+A)uv \tag{1}$$

defined on $V = H^1(-1,1)$. Assuming $A = 10$ show that this form is bilinear, symmetric, coercive ($V-$coercive), and continuous. (For the latter two conditions determine the constants of continuity and coercivity). For what values of A do these properties still hold?

(B) Given $f(x) = x^2$, determine the boundary value problem that corresponds to: find $u \in V$

$$a(u,v) = (f,v), \quad \forall v \in V. \tag{2}$$

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2. L^2 error estimate for piecewise polynomials of degree $k = 5$

In class we have proved that $\| u - u_h \|_{L^2} = O(h^2)$ when using piecewise linears. Carry out the proof when $V_h = \{v \in V, v|_E \in \mathbf{P}_5; \forall E \in T_h\}$ to determine the order k in $\| u - u_h \|_{L^2} = O(h^k)$. What is k ?

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3. Poincaré-Friedrichs constant (extra credit)

Consider $\Omega = (0,1)$. One can prove that the best (smallest) constant C_{PF} in

$$\| v \|_{L^2} \leq C_{PF} \| v \|_{H_0^1}, \quad \forall v \in H_0^1 \tag{3}$$

can be found by considering the eigenvalue problem

$$-u'' = \lambda u, \quad x \in (0,1) \tag{4a}$$

$$u(0) = u(1) = 0 \tag{4b}$$

Demonstrate the connection between (which?) eigenvalue λ in (4) and C_{PF}. **Hint:** start with the variational formulation of (4).

Show how you would find C_{PF} and λ numerically. (You can use `fem1d_2017.m`).