The Riesz map of a Hilbert space H to its dual H' is given by

$$\mathcal{R}u(v) = (u, v)_H, \quad u, v \in H.$$

Let G be a bounded domain in $\mathbb{R}^N$.

1. Suppose $a \in L^\infty(G)$ with $0 < c \leq a(x)$ a.e. on G. On $H = L^2(G)$, show that $(u, v)_a = \int_G a(x)u(x)v(x)\,dx$ is an equivalent scalar product, and denote the corresponding Hilbert space by H_a. Characterize the Riesz map $\mathcal{R}_a$ of H_a with generalized functions. (Note that $H_a \subset C_0^\infty(G)^*$ and $H_a' \subset C_0^\infty(G)^*$.)

2. For the Sobolev space $H_0^1(G)$ with the (equivalent) scalar product $(u, v)_1 = \int_G \boldsymbol{\nabla} u(x) \boldsymbol{\nabla} v(x)\,dx$, characterize the Riesz map with generalized functions.