NAME:____________________________ **ID#:**__________________________

MATH 480, Spring 2005 **TEST #2**

FOR EACH PROBLEM FIND ALL SOLUTIONS. SHOW ALL ESSENTIAL STEPS.

1.

(a) Show that there is more than one solution of the initial-value problem

$$\dot{u}(t) = u^{\alpha}(t), \quad u(0) = 0\,,$$

on $t \geq 0$, where the number α satisfies $0 < \alpha < 1$.

(b) Show that there is at most one solution of the initial-value problem

$$\dot{u}(t) = -u^{1/3}(t), \quad u(0) = u_0\,,$$

on $t \geq 0$.

2.

(a) Show that if the function $f(t, s)$ satisfies

$$(f(t,u) - f(t,v))(u - v) \leq K(u - v)^2, \tag{1}$$

then there is *at most one* solution of the initial-value problem

$$\dot{u}(t) = f(t, u(t)), \quad u(0) = u_0\,,$$

on $t \geq 0$.

(b) Describe in terms of *monotonicity properties* the functions $f(t, u)$ for which the condition (1) holds.

(c) Show that if

$$|f(t,u) - f(t,v)| \leq K|u - v|, \tag{2}$$

for some constant K,then there is *at most one* solution of the initial-value problem

$$\dot{u}(t) = f(t, u(t)), \quad u(0) = u_0\,,$$

on any interval of the form $|t| \leq T$.

(d) Describe in terms of *growth-rate properties* the functions $f(t, u)$ for which the condition (2) holds.

3. Let $u(t)$ and $v(t)$ be solutions of the initial value problems

$$\dot{u}(t) + \sin(u(t)) = 0, \quad u(0) = u_0\,,$$

$$\dot{v}(t) + \sin(v(t)) + \frac{1}{10}\sin(10v(t)) = 0, \quad u(0) = u_0\,.$$

Estimate the difference $|u(t) - v(t)|$ for $0 \leq t \leq T$ in terms of $u_0 - v_0$.