

Let  $G$  be a region in the plane. The heat conduction problem is to find  $u = u(x, y, t)$  which satisfies the initial-boundary-value problem

$$\begin{aligned}\frac{\partial u}{\partial t} - \alpha^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) &= 0, & (x, y) \in G, \quad t > 0, \\ u &= 0 \text{ on the boundary } \partial G, \\ u(x, y, 0) &= u_0(x, y).\end{aligned}$$

1. Let  $G = (0, a) \times (0, b)$ . Find the solution if
  - (a)  $u_0(x, y) = 1$ ,
  - (b)  $u_0(x, y) = x$ ,
  - (c)  $u_0(x, y) = xy$ .