MTH614: Exercises

1. Let X, Y be normed linear spaces and $T \in \mathcal{L}(X, Y)$.
Show that the second dual $T'' \in \mathcal{L}(X'', Y'')$ is an extension of T.

Let X be a Banach space and Y a closed subspace of X.

2. Show the quotient space X/Y is a Banach space and that the quotient map $\pi : X \to X/Y$ is a continuous surjection.

3. Show that the dual of the injection $\iota : Y \to X$ is the restriction $r : X' \to Y'$, $r(f) = f|_Y, f \in X'$. Find $\mathrm{Rg}(r)$ and $\mathrm{Ker}(r)$.

4. Show that the dual of the quotient map $\pi : X \to X/Y$ is an injection $\pi' : (X/Y)' \to X'$ and its range is $\mathrm{Rg}(\pi') = Y^o$.