

Problem 1

Find the *eigenfunction expansion* of the solution of the initial-boundary-value problem

$$\begin{aligned}u_t(x, t) + u_x(x, t) - \alpha^2 u_{xx}(x, t) &= f(x, t), & 0 < x < \ell, & \quad t > 0, \\u(0, t) = 0, \quad u(\ell, t) &= 0, & & \quad t > 0, \\u(x, 0) &= u_0(x), & 0 < x < \ell.& \end{aligned}$$

Hint: Choose appropriate $\beta > 0$ so that $w(x, t) = e^{-\beta x}u(x, t)$ satisfies a simpler problem.

Problem 2

Find the orthogonal complement $C_0^\infty(a, b)^\perp$ in the Hilbert space $H^1(a, b)$ for each of the three cases:

- (i) (a, b) is a bounded interval;
- (ii) $(a, b) = (0, \infty)$;
- (iii) $(a, b) = (-\infty, \infty)$.

Due: Tuesday, 14:00, December 6.