Let $\gamma: W^{1,p}(G) \to L^p(G)$ be the *trace map* and denote its *range* by $\mathbf{B}$. Let $b \in \mathbf{B}$ and consider the non-homogeneous Dirichlet problem

$$-\sum_{j=1}^{n} \partial_j a(\partial_j u(x)) = 0 \text{ in } G, \quad u(s) = b(s) \text{ on } \partial G.$$

Here the function $a(\cdot)$ is given by $a(s) = |s|^{p-1} sgn(s)$ with $1 < p < \infty$. By means of a translation, this can be rewritten as follows. Let $w = u - u_b$, where the function $u_b \in W^{1,p}(G)$ is chosen with $\gamma(u_b) = b$. Then w is characterized by

$$w \in W_0^{1,p}(G): \quad -\sum_{j=1}^{n} \partial_j a(\partial_j (w + u_b)) = 0 \text{ in } W_0^{1,p}(G)'.$$

Thus, if we define $\mathbf{V} = W_0^{1,p}(G)$ and

$$\mathcal{A}(w)(v) = \int_G \sum_{j=1}^{n} a(\partial_j (w + u_b))\, \partial_j v \, dx\,, \qquad w,\ v \in \mathbf{V}$$

then $\mathcal{A}: \mathbf{V} \to \mathbf{V}'$.
Exercise. Show that $\mathcal{A}$ is strictly-monotone, continuous, bounded and coercive.
Corollary. For each $b \in \mathbf{B}$, there is a unique

$$u \in W^{1,p}(G): \ \gamma(u) = b \text{ and } \int_G \sum_{j=1}^{n} a(\partial_j u) \partial_j v \, dx = 0 \text{ for all } v \in W_0^{1,p}(G).$$

Next we define $\mathcal{B}: \mathbf{B} \to \mathbf{B}'$ as follows. Let $b,\ \tilde{b} \in \mathbf{B}$ be given. Let $u \in W^{1,p}(G)$ be given as above. Let $v \in W^{1,p}(G)$ with $\gamma(v) = \tilde{b}$ and define

$$\mathcal{B}(b)(\tilde{b}) = \int_G \sum_{j=1}^{n} a(\partial_j u) \partial_j v \, dx\,.$$

Exercise. Show that the preceding integral is independent of the choice of $v \in W^{1,p}(G)$ with $\gamma(v) = \tilde{b}$, so this defines a function $\mathcal{B}: \mathbf{B} \to \mathbf{B}'$ as desired.
Exercise. Show that if for a $b \in \mathbf{B}$ the corresponding $u \in W^{1,p}(G)$ constructed above happens to be smooth, then

$$\mathcal{B}(b)(\tilde{b}) = \int_{\partial G} \sum_{j=1}^{n} a(\partial_j u) \nu_j \tilde{b} \, ds, \quad \tilde{b} \in \mathbf{B},$$

where ν is the unit outward normal on ∂G. Thus, $\mathcal{B}(b) = \sum_{j=1}^{n} a(\partial_j u)\nu_j$ when the function u is smooth.