DEFORMABLE POROUS MEDIA

The Darcy Flow Model

fluid content: $\eta = c_0\, p$ *fluid pressure*: $p = p(x,t)$

c_0 is the combined effect of *porosity* and *compressibility*

flux: $q_i = -k_{ij}\partial_j p$ **Darcy's law**

where k_{ij} is the *hydraulic conductivity*

Conservation of fluid mass: $\dot{\eta} + \partial_i q_i = F$

where $F(x,t)$ is the fluid source

Diffusion Equation: $c_0\,\dot{p} - \partial_i k_{ij}\partial_j p = F(x,t)$

The Navier System

strain: $\varepsilon_{kl} = \frac{1}{2}(\partial_k u_l + \partial_l u_k)$ *solid displacement*: $\mathbf{u} = \mathbf{u}(x,t)$

stress: $\sigma_{ij} = a_{ijkl}\varepsilon_{kl}(\mathbf{u})$ **Hooke's law**

where a_{ijkl} is the *resistance*, the inverse of *compliance*

Momentum Balance: $\rho\ddot{u}_i = \partial_j\sigma_{ij} + f_i\,,\ 1 \le i \le 3$

where $\mathbf{f}(x,t)$ is the body force

Wave Equation: $\rho\ddot{\mathbf{u}} - (\lambda+\mu)\nabla(\nabla\cdot\mathbf{u}) - \mu\Delta\mathbf{u} = \mathbf{f}(x,t)$

The Biot Model

fluid content: $\eta = c_0\, p + \nabla \cdot \mathbf{u}$ *dilation effect*

stress: $\sigma_{ij} = a_{ijkl}\varepsilon_{kl}(\mathbf{u}) - p\,\delta_{ij}$ **Terzaghi's law**

$+\ \mu^* \nabla \cdot \dot{\mathbf{u}}\,\delta_{ij}$ *secondary consolidation*

Fully Dynamic Poro-Elasticity System (Biot = Navier + Darcy)

... a system of mixed *hyperbolic-parabolic type*

$$\rho \frac{\partial^2 \mathbf{u}(t)}{\partial t^2} - \mu^* \nabla \frac{\partial}{\partial t}(\nabla \cdot \mathbf{u}(t)) + \mathcal{E}(\mathbf{u}(t)) + \nabla p(t) = \mathbf{f}(t)\,,$$

$$c_0 \frac{\partial p(t)}{\partial t} + A(p(t)) + \nabla \cdot \frac{\partial \mathbf{u}(t)}{\partial t} = h(t)\,,$$

where

$$\mathcal{E}_0(\mathbf{u}) \equiv -(\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}) - \mu \Delta \mathbf{u}\,, \qquad A_0(p) \equiv -\nabla \cdot (k \nabla p)\,.$$

The Quasi-Static System

$$-\mu^* \nabla \frac{\partial}{\partial t}(\nabla \cdot \mathbf{u}(t)) + \mathcal{E}(\mathbf{u}(t)) + \nabla p(t) = \mathbf{h}(t)\,,$$

$$c_0 \frac{\partial p(t)}{\partial t} + Ap(t) + \nabla \cdot \frac{\partial \mathbf{u}(t)}{\partial t} = h(t)\,.$$