

Math Faculty as Partners in Teaching a non-Euclidean Geometry Course for K-12 Teachers

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Summary of Partnership

- Curriculum development
 - Course implementation
 - Course expansion and revision
 - Faculty professional development and growth
 - Dissemination efforts
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Geometry for K-12 Teachers

- Oregon Mathematics Leadership Institute (OMLI)
 - NSF-funded partnership project— OSU/PSU/TDG/10 OR school districts (NSF/EHR-0412553; ODE/Oregon ESEA Title II-B MSP)
 - One of 6 mathematics courses
 - Aimed at deepening geometry content knowledge of K-12 teachers
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Non-Euclidean Geometry

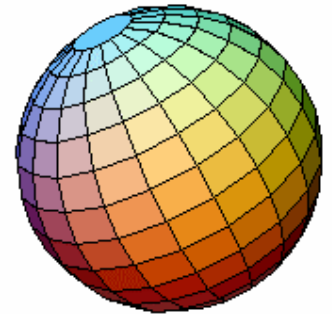
- OMLI district leaders recommend non-Euclidean geometry
 - Spherical geometry
 - Taxicab geometry
 - “Comparing Different Geometries”
 - Intense: 15 two-hour sessions
 - Team-taught
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The Geometry Team

- Research mathematician with interest in mathematics education
 - Master teacher
 - Mathematics education specialist from a teaching university
 - 2 college instructors with varied teaching experience
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Curriculum Highlights

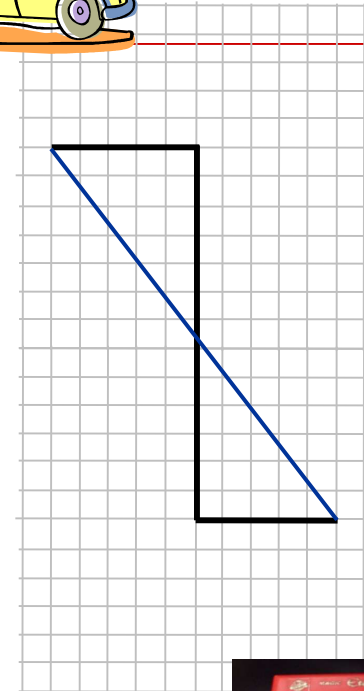
- Unit on spherical geometry
 - Lines
 - Parallel and perpendicular lines
 - Set of points equidistant from 2 different points
 - Common perpendicular
 - Polygons: triangles, “squares”
 - Circles



Curriculum Highlights Continued



- Unit on Taxicab geometry
 - Distance
 - Midpoints
 - Sets of points equidistant from two given points
 - Squares
 - Circles and π
 - Triangles and congruence



Preparation for Delivery

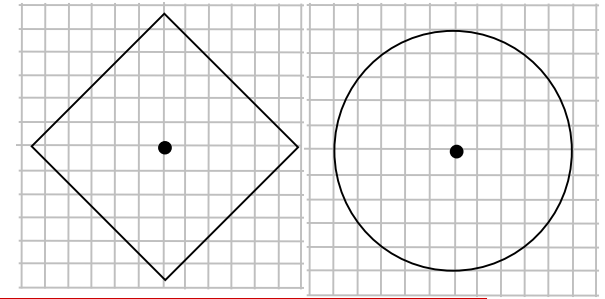
- *Teachers Development Group* workshops and readings:
 - “Best Practices”
 - Cooperative Learning
 - Promoting Discourse in the Classroom
 - Lesson plans
 - Daily debrief sessions
 - Reflection and revision
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OMLI's Role

- Numerous opportunities to meet
 - Year-round communication
 - Retreats
 - Facilitation of exchange of ideas
 - In person
 - Via e-mail
 - Development of group dynamics
 - Cooperative division of labor
 - Embracing similarities and differences in style
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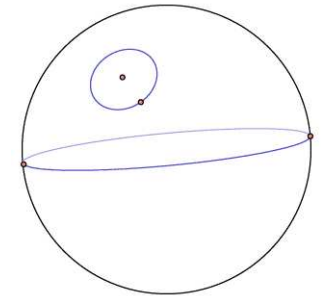
Implementation

- Building comparison charts
 - Spherical with Euclidean geometry
 - Taxicab with Euclidean geometry
- Group projects
 - Capstone experience: extension of ideas
 - Presentations and posters

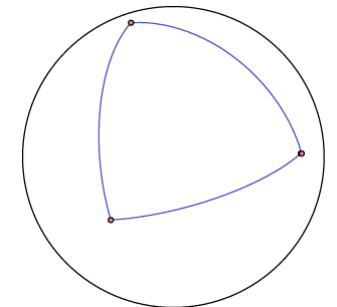
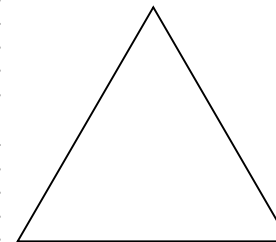
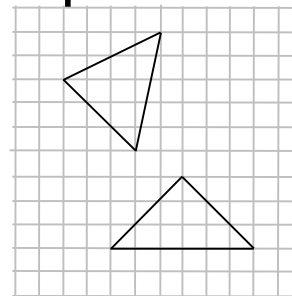


$\pi=4$

$\pi \sim 3.14$



$\pi \in [2, 3.14]$



Equilateral Triangles

Pedagogical Lessons

- More and better mathematics discourse and learning:
 - Rich mathematical tasks
 - Orchestrated cooperative learning
 - Effective facilitation
 - Comparison: relating, connecting, multiple representations
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Expansion and Revision

- ❑ Focus on undefined terms, axioms, and precise definitions
 - ❑ Switch of topic order and introduce new manipulative aids: Etch-a-sketch
 - ❑ More sophisticated cooperative learning strategies: jigsaw puzzle, differentiated tasks
 - ❑ Role of writing: from writing reflections, to prompts, to exit cards
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Faculty Growth

- Challenges and triumphs of team teaching
 - Prepare, share, collaborate
 - Draw on each other's strengths
 - Stay flexible
 - Learn and reflect
 - Lesson study
 - Open door policy
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Faculty Impact

- Threefold impact on own classrooms
 - Mathematics discourse
 - Mathematical tasks
 - Group work
 - Facilitation skills
 - Projects: capstone experiences
 - Assessment: reflective, peer, rubrics
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Mathematical Tasks

- ❑ Multiple entry points
 - ❑ Group-worthy
 - ❑ Applied, hands-on nature
 - ❑ More open ended
 - Use an Etch-a-Sketch to explore the notion of Taxicab geometry distance by measuring the length of the diagonal line on the Etch-a-Sketch by turning one knob at a time.
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Group Work

- Awareness of equity issues
 - Heterogeneous groups
 - Careful selection and rotation
 - Roles
 - Judicious Orchestration
 - Size of groups
 - Huddles
 - Protocols
 - Advanced collaborative learning strategies
 - Jigsaw
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Facilitation Skills

- Better questioning techniques
 - Observing and sequencing
 - from simple to complex
 - from kernels of idea to its development
 - handling student mistakes by letting groups respond
 - Connecting student ideas better
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Focus on Students' Thinking

- ❑ "Less is more."
 - ❑ Listen, observe, take notes.
 - ❑ Use student feedback to adjust teaching.
 - ❑ Focus on justification and sense-making at all times.
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Dissemination Efforts

- ❑ Entire set of curriculum materials is ready for publication through *Teachers Development Group*
 - ❑ Article accepted in The Montana Mathematics Enthusiasts Monograph on Discourse (pedagogy)
 - ❑ Article in preparation to be submitted to School Science and Mathematics (content)
 - ❑ "Dare to Compare? Introduction to Using Comparison Charts in the Mathematics Classroom" appeared in May/June 2008 TOMT
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